Purpose: In this problem set, we will investigate the tip of the rational function iceberg. In particular, we will work to understand the domains of rational functions.

Definition: A **Rational Function** is a ratio of polynomials in which the denominator is nonzero. We often write them in the form

$$f(x) = \frac{A(x)}{B(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + a_0}.$$

We have a lot of questions about rational functions:

- What is the domain of a rational function?
- What is the end behavior of a rational function?
- What are the x and y-intercepts of a rational function?
- What does the graph of a rational function look like?

1. Find the domain of $R(x) = \dfrac{1}{x^2 - 1}$.

Definition: A _____ of a graph is a vertical line that the graph approaches but never touches.

2. Find the domain and vertical asymptotes of $f(x) = \dfrac{10}{(x-5)(x+4)(x-2)}$. Once you've done so, have someone in your group graph the function in Desmos and sketch the graph below.

3. Find the domain and vertical asymptotes of $g(x) = \dfrac{10}{(x-5)^2(x+4)(x-2)}$. Once you've done so, have someone in your group graph the function in Desmos and sketch the graph below.

4. Compare and contrast the graphs of the previous two rational functions, $f(x)$ and $g(x)$.

5. Find the domain and vertical asymptotes of the graph of $y = \dfrac{x+1}{x^2-1}$. Once you've done so, have someone in your group graph the function in Desmos and sketch the graph below.

Definition: A _____ of a graph is a point that the graph approaches but never touches.

6. Find the vertical asymptotes and holes of the graph of $y = \dfrac{(x-5)(x+4)}{(x-5)^2(x+4)(x-2)}$. Once you've done so, have someone in your group graph the function in Desmos and sketch the graph below.